Ask a Scientist Pi Day Puzzle Party 3.14 2015

Wes Carroll bodsat.com & **Juliana Gallin** askascientistsf.com

1. CASH REGISTER

A barber starts the day with a certain amount of money in his cash register. After he finishes the first haircut of the day, he says to the customer, "Double the amount of money currently in the cash register, using money from your pocket. Then take out $20." He says the same to the second third and fourth customers. But, when he gets to the 5th customer, the customer says, "I can't do that. You're broke!"

QUESTION: How much did the barber start with in the register?

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2. TWO FERRYBOATS

Two ferryboats start at the same instant from opposite sides of a river, traveling across the water on routes at right angles to the shores. Each travels at a constant speed, but one is faster than the other. They pass at a point 720 yards from the nearest shore. Both boats remain in their slips for 10 minutes before starting back. On the return trips they meet 400 yards from the other shore.

QUESTION: How wide is the river?

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3. ABC PASSCODE

Juliana is trying to open a lock whose code is a sequence that is three letters long, with each of the letters being one of A, B or C, possibly repeated. The lock has three buttons, labeled A, B and C. When the most recent 3 button-presses form the string, the lock opens.

QUESTION: What is the minimum number of total button presses Juliana needs to be sure to open the lock?

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4. THE FISHING ROD

One late summer day, young Wes (age 11) heard about a big sale downtown at Tacklebusters. After school, he took the bus downtown and bought a new fishing rod. It was 5 feet long and just the model he'd been wanting. He headed back to the bus stop excited to try it out on the weekend. Sadly, the bus driver wouldn't let him on the bus. "Sorry kid, but City Ordinance 132G/F29 prohibits carrying anything longer than 4 feet on city buses." Wes dejectedly walked off the bus. He didn't know what to do. His parents were at work and it was a 5 mile walk home. Suddenly, he had an inspiration. He ran back to the store, came back with the same fishing rod, and caught the next bus home with his new rod.

Note: He did not bend, break or disassemble the rod in any fashion. Nor did he bribe the next bus driver.

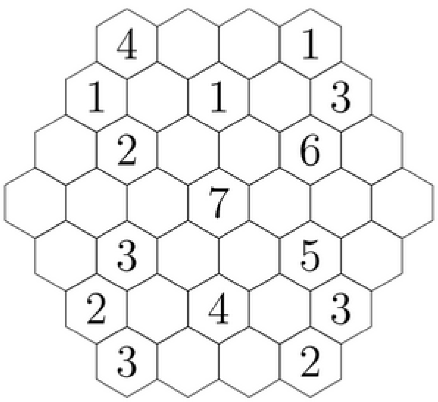
QUESTION: What did Wes do to get on the bus?

5. HONEYCOMB NUMBERS

In the grid shown here and on your answer sheet, fill in each empty space with a number, such that after the grid is completely filled in, the number in each space is equal to the smallest positive integer that does not appear in any of the touching spaces.

Note: a pair of spaces is considered to touch if they both share a vertex.

Please copy your final answers to the grid on the answer sheet.



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6. GREEN-EYED BROTHERS

If you happen to meet two of the Carlin brothers (this assumes that the two are random selections from the set of all the Carlin brothers), it is exactly an even-money bet that both boys will be green-eyed.

QUESTION: What is the total number of green-eyed Carlin brothers?

For bonus puzzle karma (though not actual points) submit one or more additional correct answers along with the smallest.

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7. SEVEN MINUS NINE

Although this equation is certainly false on its face, you can make it true by replacing each letter with a corresponding digit. In fact, this puzzle can be solved two different ways.

QUESTION: What is EIGHT plus EIGHT? (Since there are two different solutions, there are two different EIGHTs.)

S E V E N

- N I N E

= E I G H T

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8. BLANK AND NORMAL DICE

You have one die with normal numbers and another die that's completely blank.

QUESTION: How should you number the blank die so that with every roll (of both dice) there is an equal chance of getting every number from 1 - 12?

TIEBREAKER:

THE ENIGMATIC SEQUENCE

I invite you to consider the following sequence of numbers: 1, 3, 7, 13, 21, ...

a) Name the the 600th member of the series

or

(d) The smaller of the two consecutive members that are exactly 1000 apart.

SOLUTION:

The nth member of this sequence is n times [n-1], + 1.

(a) The 600th member of the series is 600 * 599 + 1 = 359401.

(d) The smaller of the two consecutive members that are exactly 1000 apart is 249501
(500 * 499 + 1, which is exactly 1000 less than 250501, which in turn is equal to 501 * 500 + 1).

SOLUTIONS

→

1. CASH REGISTER

$18.75

($18.75 $17.50 $15 $10 $0)

2. TWO FERRYBOATS

When the two ferry boats meet for the first time, the combined distance travelled by the boats is equal to the width of the river. When they reach the opposite shores, the combined distance travelled is twice the width of the river and when they meet for the second time, the combined distance is three times the width of the river.

Since both boats are moving at a constant speed for the same period of time, it follows that each boat has gone three times as far as when they first met (after they had travelled a combined distance of one river's width). Since the white boat, (slow boat), had travelled 720 metres before the first meeting, its total distance at the time of the second meeting must be 3 x 720 or 2160 metres. This is 400 metres more than the width of the river, so we must subtract 400 from 2160 to obtain 1760 metres as the width of the river.

3. ABC PASSCODE

The answer is 29. This can be achieved with the following sequences of presses:

AAACCCBCCACBBCBACABCAABBBABAA

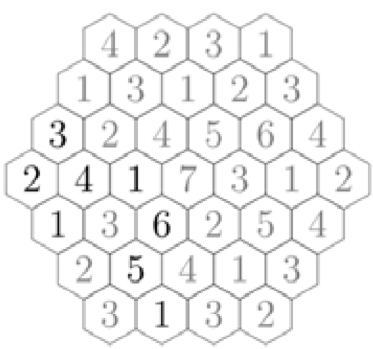
There are 3 x 3 x 3 = 27 different strings of three letters with each letter being one of A, B or C. All 27 of these strings appear consecutively in the above sequences of presses.

Each press of the button corresponds to attempting at most one more string, namely the one formed by the previous three presses. No string can be attempted after the first two presses. Therefore, the first time that all 27 strings can be tried on the lock is after 27+2=29 presses of the button.

4. THE FISHING ROD

Wes bought a box whose diagonal fits the rod, but whose sides fall within the bus regulations. (3x3x3 is more than big enough to fit the rod across the body diagonal)

5. HONEYCOMB NUMBERS



6. GREEN-EYED BROTHERS

There are probably three green-eyed Carlin brothers and four brothers altogether.

If there are n boys, of which g are green-eyed, the probability that two chosen at random are green-eyed is: (g(g-1)) / (n(n-1)).

We are told that this probability is 1/2, so the problem is one of finding integral values for g and n that will give the above expression a value of 1/2. The smallest such values are n=4, g=3 (i.e. 3 of 4). The next highest values are n=21, g=15 (15 of 21), but it is extremely unlikely that there would be as many as 21 brothers, so four brothers, three of them green-eyed, is the best guess.

(The next solutions are 85 of 120, 493 of 697, 2871 of 4060, 16731 of 23661, and 97513 of 137904.)

7. SEVEN MINUS NINE

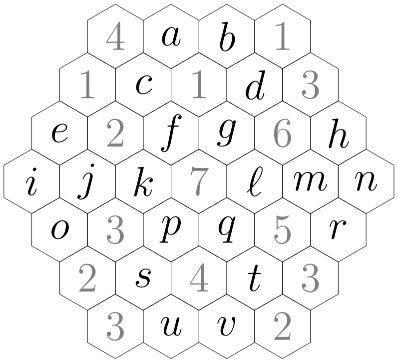
EIGHT + EIGHT = 64255

1	2	3	4	5	6	7	8	or	1	2	3	4	5	6	7	8
E	S	T	N	V	I	H	G	or	V	T	G	E	S	N	I	H

8. BLANK AND NORMAL DICE

The other die gets three blanks (or zeros) and three sixes. This way there are three ways to add up to every number from 1-12.

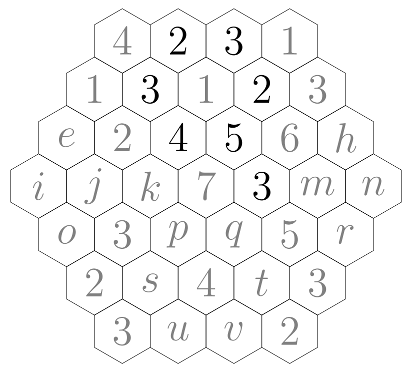
Solution



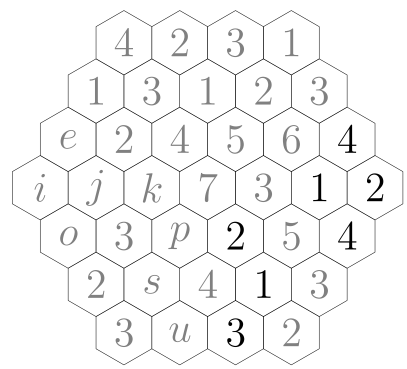
We label the cells a through v lexicographically as in the diagram to the left.

The 4 in row 1 must touch a 2, but the 2 can't be c because c already touches a 2. Therefore, $a = 2$. The 4 must also touch a 3, so $c = 3$. Finally, b must be at least 3, but can't be greater than 3 because there is no way to place a 3 in a neighboring space. Therefore, $b = 3$.

The 6 in the third row must touch a 5. That 5 cannot be ℓ or m because those each neighbor the 5 in the fifth row. The 5 also cannot be d or h because there are not enough empty spaces for d or h to neighbor all of 1, 2, 3, and 4. Therefore, $g = 5$. Now the three vacant spaces touching g must have the numbers 2, 3, and 4. Since f touches a 2 and a 3, it must be the 4. Since d touches a 3, it must be the 2. This leaves $\ell = 3$. The progress so far is summarized in the diagram to the right.



Consider the 4 in the sixth row. It must touch a 3, but the only space surrounding it that does not touch a 3 is v, so $v = 3$. The 2 in row 7 must touch a 1, so $t = 1$. Next, consider $\ell = 3$. It must touch a 1 and a 2, so m and q must have a 1 and 2 between them. But q touches a 1, so $q = 2$ and $m = 1$. To finish the right edge, consider the 5 in the fifth row. It must touch a 4, so $r = 4$. We can quickly determine that $n = 2$ and $h = 4$ from here.



Our progress so far is summarized in the diagram to the right. Next, the 7 in the center must touch the six numbers 1 to 6 once each. We already found the numbers 2 through 5, so k and p are 1 and 6 between them. Notice that regardless of which of the two is the 6, the remaining vacant space it touches (either j or s) must be a 5. In particular, this means each of j and s is either 5 or neighbors a 1, so neither can be a 1. Since the 3 in row 7 must touch a 1, that 1 must be u. Then $s = 5$, since it now touches 1 through 4, but $p \neq 5$.

The 2 in row 6 must touch a 1, so $o = 1$. All of the vacant spaces around i touch a 2, so i is at most 2. It is also at least 2 since it touches a 1, so $i = 2$. By similar reasoning, e is a 3 and j is a 4. Since j is not a 5, the 6 must be p, and the 1 goes in k. This completes the grid. The only solution is the one shown below.